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Statistical Estimation of Economic Life for Aircraft Structures

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An analytical methodology for the statistical prediciton of the economic life of advanced aircraft structures is presented. The approach allows for the determination of the economic life based on either one of the following criteria: 1) a rapid increase of the number of crack damages exceeding the economic repair crack size, and 2) a rapid increase of the maintenance cost, including the costs of inspection and repair. The formulation is general enough for practical applications. While the inspection and repair maintenance procedure has a significant impact on aircraft structural reliability and safety, its effect on the economic life is shown to be limited. Numerical examples are worked out to demonstrate the application of the methodology.

I. Introduction

CURRENT U.S. Air Force structural integrity (MIL-STD-1530A) and durability design specifications (MIL-A-8866B) require that airframe components be designed such that the economic life be analytically predicted. The conventional fatigue analysis, while capable of estimating design life, does not lend itself to predicting the economic life, nor is it capable of providing a definition of economic life. The damage tolerant design as specified in MIL-A-83444 requires that a 0.005-in. crack is assumed to be presented in fastener holes to simulate the possible existence of material and manufacturing defects. In predicting the economic life, the entire crack population should be taken into account, and hence the statistical approach is essential. Literature describing the current practice regarding crack growth damage accumulation with applications to structural safety, durability, damage tolerance, and reliability can be found in Refs. 1-3.

The cost of maintenance, including the costs of inspection, repair, rework, replacement, etc., for aircraft structures in order to fulfill the requirements of safety, durability, damage tolerance, and reliability is of practical importance. It is well-known that after a certain service life, referred to as the economic life, either the cost of maintenance or the number of cracks exceeding the economical repair crack size increases so rapidly that the durability requirement cannot be satisfied. In an attempt to demonstrate statistically the existence of the economic life, an exploratory investigation has been made in which restrictive limitations have been imposed for simplicity.[4]

An analytical methodology capable of quantitatively predicting the economic life of advanced metallic aircraft structures has been developed. The methodology developed is based on sound analytical and statistical approaches. It accounts for various service conditions, such as any type of initial fatigue quality, crack growth damage accumulation, loading spectra, material/structural properties, usage change, inspection and repair maintenance, etc. Hence, the methodology presented herein is general enough for practical applications. The formulation allows for the determination of the economic life using either one of the following two criteria: 1) a rapid increase of the number of crack damages exceeding the economic repair crack size, and 2) a rapid increase of the maintenance cost. The economic repair crack size a_e is defined as the crack size below which the least expensive repair procedure can be used, such as reaming the fastener holes to the next hole size. a_e is usually between 0.03 and 0.05 in. depending on the location and the fastener hole size.

The initial fatigue quality of a durability critical component is characterized by the equivalent initial flaw size (EIFS),[5-7] while the input initial quality to the present analysis is the test result of the time to crack initiation (TTCI) which is physically observable. The input data of TTCI, which are statistically characterized by the three-parameter Weibull distribution, are transformed through backward extrapolation herein by use of a crack propagation law to determine the distribution of the EIFS.[2,8-10]

A maintenance program can be implemented in service where inspections and repairs are performed at scheduled intervals. The statistical uncertainty of the NDI procedure has been accounted for. When a cracked detail is detected and repaired, its fatigue strength is renewed in the sense that its crack size is identical to that of the EIFS prior to service.

The entire population of the EIFS, as specified by its statistical distribution, is then subjected to service loading spectra consisting of possibly various different missions. The crack growth damage accumulation under spectrum loading can be computed by use of a general computer program[2,9-13] or test results including that of the fractography. Thus, the distribution of the crack size is shifted and changed continuously in service due to crack propagation. It is also subjected to truncation and modification after each inspection and repair. The distribution of the crack size at any service time is then derived, and the percentage of cracked details exceeding any crack size with certain probability and confident levels has been computed as a function of service time.

It is demonstrated numerically that the percentage of cracks exceeding the economic repair crack size (with any probability and confident levels) increases rapidly after a certain service time, thus determining the economic life of the component. While the inspection and repair maintenance procedure has a significant impact on the safety and reliability of aircraft structures,[14-18] its effect on the economic life of a component is shown to be limited.

II. Formulation

A. Crack Exceedance and Maintenance Cost

A durability critical component is divided into m stress regions. In each stress region, the maximum stress level at every location or detail (e.g., fastener hole) is approximately

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the same, whereas the maximum stress level varies from one stress region to another. Let N_i be the total number of details in the ith stress regions. If the maximum stress varies from one location to another, then $N_i = 1$.

In the ith stress region, let $N(i,\tau)$ represent the total number of details having a crack size exceeding x_l at any service time τ. Each detail, such as the fastener hole, may have multiple cracks, and the crack of a detail refers herein to the largest crack. Hence, each detail has one crack (the largest one).

It is reasonable to assume that the crack growth at each detail is not interfered by the cracks in its neighboring details and that the crack damage accumulation of each detail is statistically independent. Then, the distribution of $N(i,\tau)$ can be shown to follow the binomial distribution.

$$P[N(i,\tau)=n] = \binom{N_i}{n} p^n(i,\tau)\left[1-p(i,\tau)\right]^{N_i-n} \quad (1)$$

in which $P[N(i,\tau)=n]$ denotes the probability that the total number of details having a crack size exceeding x_l in the ith stress region $N(i,\tau)$ is equal to n, and $p(i,\tau)$ is the probability that a detail in the ith stress region will have a crack size greater than x_l at the service time τ.

The average value $\overline{N}(i,\tau)$ and the variance $\sigma_N^2(i,\tau)$ of $N(i,\tau)$ can be obtained from Eq. (1) as

$$\overline{N}(i,\tau) = N_i p(i,\tau), \qquad \sigma_N^2(i,\tau) = N_i p(i,\tau)\left[1-p(i,\tau)\right] \quad (2)$$

If N^* is the total number of details in the entire component and $L(\tau)$ indicates the total number of details having a crack size larger than x_l at τ, then one has

$$L(\tau) = \sum_{i=1}^{m} N(i,\tau), \qquad N^* = \sum_{i=1}^{m} N_i \quad (3)$$

Thus, the average value $\overline{L}(\tau)$ and the variance $\sigma_L^2(\tau)$ of $L(\tau)$ are obtained, respectively, by

$$\overline{L}(\tau) = \sum_{i=1}^{m} \overline{N}(i,\tau), \qquad \sigma_L^2(\tau) = \sum_{i=1}^{m} \sigma_N^2(i,\tau) \quad (4)$$

in which $\overline{N}(i,\tau)$ and $\sigma_N^2(i,\tau)$ are given by Eq. (2).

The component may undergo a scheduled inspection and repair maintenance procedure at T_1, T_2, T_3...in service with service intervals τ_1, τ_2, τ_3...as shown in Fig. 1. It is assumed that the cracked details detected during inspection will be repaired. Let $g_j(x)\mathrm{d}x$ be the probability of detecting (or repairing) a crack size between x and $x+\mathrm{d}x$ at the jth inspection. Then, the average maintenance cost, consisting of the costs of inspections and repairs, in the service interval $(0, T_{I+1})$, denoted by $C(\overline{I})$, is

$$\overline{C}(I) = C_1 N^* I + N^* \sum_{j=1}^{I} \int_0^\infty C_2(x) g_j(x)\,\mathrm{d}x \quad (5)$$

in which C_1 = cost of inspecting one detail, I = total numbers of scheduled inspections, $C_2(x)$ = cost of repairing one cracked detail with a crack size x. Note that $C_2(x)$ is an increasing function of the crack size x and it may be a discontinuous function of x.[4]

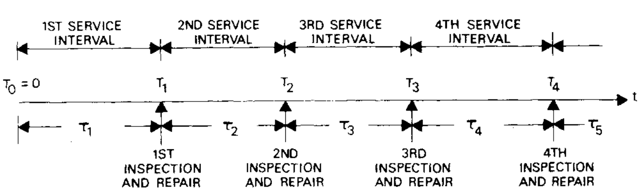

Fig. 1 Definition of service interval.

It is observed from Eqs. (1-5) that the estimation of the crack exceedance and the maintenance cost depends on $p(i,\tau)$ and $g_j(x)$. These two quantities along with the structural reliability will be derived later.

B. Initial Fatigue Quality

One of the most important factors effecting the durability analysis of a component is the initial fatigue quality which can be characterized by either the TTCI or the EIFS. The TTCI is defined as the time T at which a visible crack, say 0.03 in., occurs. Attempts have been made to characterize the statistical distribution of TTCI by use of the Weibull distribution.[19-23] Laboratory test data[7] of TTCI for fastener holes under a fighter spectrum are fitted herein by the three-parameter Weibull distribution

$$F_T(t) = P[T \le t] = 1 - \exp\{-[(t-\epsilon)/\beta]^\alpha\}; \qquad t \ge \epsilon \quad (6)$$

in which α = shape parameter, β = scale parameter, and ϵ = location parameter (lower bound). One set of test data plotted on the Weibull probability paper as circles is presented in Fig. 2a along with the fitted Weibull distribution (straight line).

One difficulty involved in using the method of TTCI is that it depends on the stress level and loading spectra. Since the maximum stress level of the loading spectra varies from one stress region of the component to another, it is not feasible to obtain test data of TTCI associated with all the possible maximum stress levels of concern.

The method of the EIFS assumed the existence of a distribution of cracks which represent defects prior to ser-

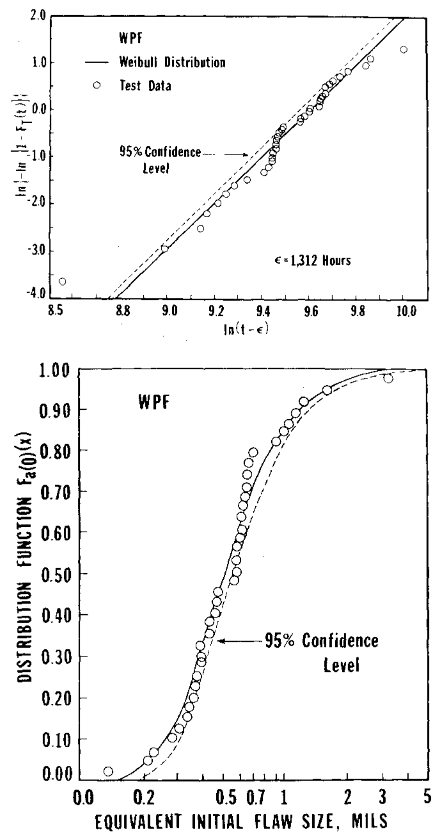

Fig. 2 Distributions of time to crack initiation and equivalent initial flaw size: a) Weibull distribution for TTCI, b) Weibull compatible distribution for EIFS.

vice.[2,5-7] Since most of the initial cracks existing in the component are not detectable by any NDI technique, the initial crack size is obtained from the TTCI results of laboratory fatigue tests. Then, the TTCI data are extrapolated backwards by use of the crack growth law to estimate their "equivalent" initial flaw size.

In the present approach, the backward extrapolation is performed using the crack growth rate equation.

$$\mathrm{d}a(t)/\mathrm{d}t = Qa^b(t) \tag{7}$$

in which Q and b are parameters depending on loading spectra, structural and material properties, etc., and $a(t)$ is the crack size at t.

The validity of Eq. (7) in the small crack size region has been verified by both the fractographic results[8] and the results using the general computer program for crack growth damage accumulation.[9,10] The theoretical justification of Eq. (7) using the miniblock approach[11-13] was made in Refs. 8 and 9.

By use of the results of the integration of Eq. (7), the distribution function of the initial crack size $a(0)$ can be derived from that of TTCI given by Eq. (1) with the result[8,9]

$$F_{a(0)}(x) = \exp\left\{-\left[(x^{-c} - a_0^{-c} - cQ\epsilon)/cQ\beta\right]^{\alpha}\right\};$$

$$\text{for } 0 \leq x \leq x_u = (a_0^{-c} + cQ\epsilon)^{-1/c} \tag{8}$$

in which $c = b - 1$ and $a_0 = 0.03$ in. is the crack size at crack initiation. Eq. (8) is referred to as the Weibull compatible distribution.

For given α and ϵ, the γ confident level for β, denoted by β_γ, is given by

$$\beta_\gamma = \beta\left[\frac{1}{2n}\kappa_\gamma^2(2n)\right]^{-1/\alpha} \tag{9}$$

in which n is the total number of test data, and $\kappa_\gamma^2(2n)$ is γ-fractile of the chi-square variate with $2n$ degree of freedom.

Data of EIFS for the same set of specimens presented in Fig. 2a are obtained from backward extrapolation using Eq. (7) and are plotted in Fig. 2b as circles, along with a solid curve representing Eq. (8). It is observed from Fig. 2b that the correlation between the EIFS and the theoretically derived Weibull compatible distribution of Eq. (8) is excellent.

Ideally, the EIFS should be independent of loading spectra, i.e., it is only a function of materials, manufacturing, and assembling processes. If so, the advantage of the method of EIFS is that the crack growth damage accumulation under different loading spectra can be calculated analytically or numerically starting from the distribution of EIFS [Eq. (8)]. However, the Weibull parameters α, β, and ϵ, [Eq. (6)] as well as $c = b - 1$ and Q [Eq. (7)] are spectrum dependent. Whether the combination of these parameters appearing in Eq. (8) will result in a distribution for the EIFS, which is spectrum dependent, remains to be investigated.

Although the Johnson distribution has been used for the distribution of EIFS,[2,4] the Weibull compatible distribution presented in Eq. (8), which is derived from the Weibull distribution of TTCI [Eq. (6)], is preferable. From the physical standpoint, it is generally agreed that the fatigue of metals is a wear-out process indicating that the failure rate increases monotonically. The Weibull distribution satisfies such a condition, and hence the distribution given by Eq. (8) for EIFS also implies the wear-out process for fatigue.[8,9]

C. Crack Growth Damage Accumulation

The entire population of the EIFS is subjected to crack propagation in service. Because of the crack geometry, the effect of loading sequence, e.g., retardation and acceleration, and others, the crack growth rate equation given by Eq. (7) does not hold for the entire region of the crack size. As a result, crack growth prediction has been carried out numerically by use of the cycle-by-cycle integration using the general computer program called CGR developed by General Dynamics. Thus, the crack size $a(t_2)$ at t_2 flight hour can be expressed in terms of $a(t_1)$, where $t_1 < t_2$ as

$$a(t_2) = a(t_1) + \Sigma\Delta a(t_j) \tag{10}$$

in which $\Delta a(t_j)$ is the crack growth increment per flight hour at t_j, where $t_1 \leq t_j \leq t_2$. The analytical crack growth curve $a(t)$ as a function of service time t is referred to as the "master curve."

Since the design loading spectra consist of many repeated flights and missions, it is reasonable to assume that the relation between $a(t_1)$ and $a(t_2)$ depends on the difference of the service time $t_2 - t_1$. Thus, only one master curve for each maximum stress level is sufficient for the determination of the crack growth damage. For the purpose of mathematical derivation later, the analytical master curve $a(t)$ can be symbolically represented by

$$a(t_1) = W[a(t_2), t_2 - t_1] \tag{11}$$

in which W is a general function representing the master curve. For instance, for a special case in which Eq. (7) is valid, one obtains the function W after integration as $a(t_1) = W[a(t_2), t_2 - t_1] = a(t_2)/[1 + a^c(t_2)cQ(t_2 - t_1)]^{1/c}$. One master curve is needed for each stress region.[9-10]

In the present analysis, the master curve can be obtained, starting from an arbitrary crack size that is smaller than the EIFS, by use of any general crack growth damage accumulation package and method,[2,3,11-13,24-28] such as CGR program. It appears that the miniblock approach proposed by Gallagher[11-13] using the method of flight-by-flight integration is most efficient for the present purpose. Two master curves for the fastener holes of 7075 aluminum specimens with no load transfer, Winslow drilled with proper drilling technique under a fighter spectrum for two maximum stress levels $\sigma_{max} = 34$ ksi and $\sigma_{max} = 28.9$ ksi, respectively, are presented in Fig. 3a. The crack growth rate $\mathrm{d}a(t)/\mathrm{d}t$ vs the crack size $a(t)$ of the master curve (Curve 1) is plotted in Fig. 2b in log scale. The fact that log $\mathrm{d}a(t)/\mathrm{d}t$ vs log $a(t)$ is a straight line in the region where $a(t) \leq 0.03$ in. indicates the validity of using Eq. (7) for backward extrapolation to obtain the distribution of EIFS presented in Eq. (8).

D. Exceedance Probability $p(i,\tau)$

Even under well-controlled laboratory conditions, the detection of a given crack size is a statistical variable and it should be specified statistically. Let $F_D(x)$ be the probability of detecting a crack size x and $F_D^*(x) = 1 - F_D(x)$ be the probability of missing it. Clearly, both $F_D(x)$ and $F_D^*(x)$ depend on the crack size x and the resolution capability of a particular NDI procedure.[14-18,21] Various functional forms for $F_D(x)$ have been suggested.[14,17]

Without Inspection and Repair Maintenance

Without scheduled inspection and repair maintenance, the probability $p(i,\tau)$ that the crack size $a(\tau)$ of a detail in the ith stress region will exceed a value x_1 at any service time τ is given by

$$p(i,\tau) = P[a(\tau) > x_1] = P[a(0) > W(x_1,\tau)]$$

$$= 1 - F_{a(0)}[y_1(\tau)] \tag{12}$$

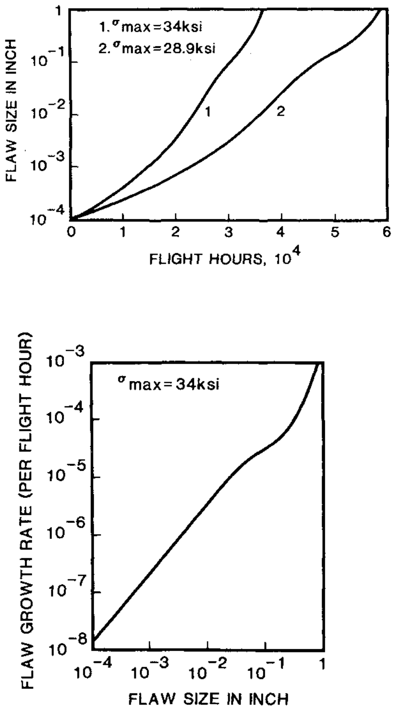

Fig. 3 Crack growth damage accumulation: a) master curve, b) crack growth rate.

in which

$$y_1(\tau) = W(x_1,\tau) \tag{13}$$

and $F_{a(0)}(x)$ is the distribution function of the EIFS $a(0)$ given by Eq. (8). In Eq. (12), the relation between $a(\tau)$ and the initial crack size $a(0)$ given by Eq. (11) with $t_1 = 0$ and $t_2 = \tau$ have been used, i.e.,

$$a(0) = W[a(\tau),\tau]$$

Thus, $p(i,\tau)$ is obtained by substituting Eq. (8) into Eq. (12)

$$p(i,\tau) = 1 - \exp\left\{-\left[\frac{y_1^{-c}(\tau) - a_0^{-c} - cQ\epsilon}{cQ\beta}\right]^{\alpha}\right\};\; y_1(\tau) \le x_u$$
$$= 0 \qquad ;\; y_1(\tau) \ge x_u \tag{14}$$

in which x_u is given by Eq. (8). The value $y_1(\tau)$ given in Eqs. (13) and (14) is obtained from the master curve as shown in Fig. 4a in which x and t are replaced by x_1 and τ. Note that $p(i,\tau)$ is a function of both x_1 and τ when Eq. (13) is used (see Fig. 4a).

Scheduled Inspection and Repair Maintenance

When the durability critical component is subjected to a scheduled inspection and repair maintenance at $T_1, T_2, T_3, \ldots$ with service intervals $\tau_1, \tau_2, \tau_3, \ldots$ as shown in Fig. 1, the solution for $p(i,\tau)$ is as follows. Let $f_{a(t)}(x)$ and $f_{a(T_j+t)}(x)$ be the probability density functions of the crack sizes $a(t)$ and $a(T_j+t)$, respectively. The probability density function $f_{a(0)}(x)$ of the initial crack size $a(0)$ is $f_{a(0)}(x) = \mathrm{d}F_{a(0)}(x)/\mathrm{d}x$. It follows from Eq. (8) that

$$f_{a(0)}(x) = (\alpha x^{c-1}/Q\beta)\Phi^{\alpha-1}\exp(-\Phi^{\alpha}) \tag{15}$$

$$\Phi = (x^{-c} - a_0^{-c} - cQ\epsilon)/cQ\beta \tag{16}$$

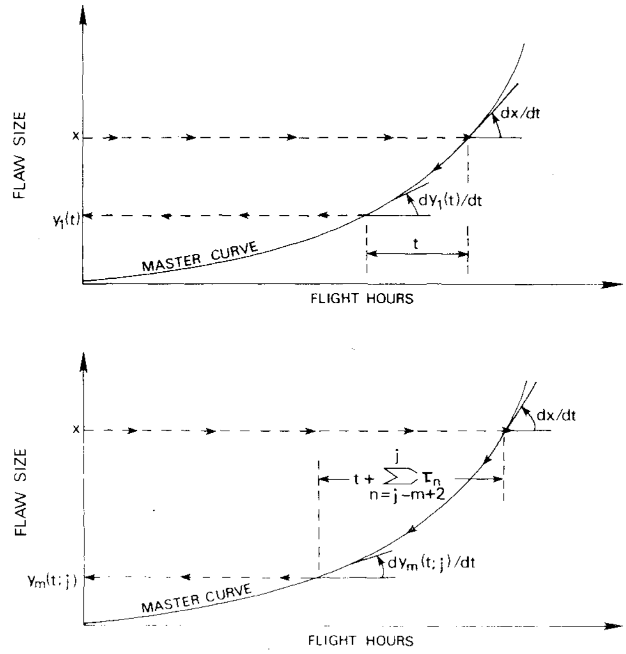

Fig. 4 Utilization of master curve.

The probability that the crack size will exceed a value x_1 at any service time $\tau = T_j + t$, Fig. 1, for $j = 0,1,2,\ldots$ is given by

$$p(i,\tau) = \int_{x_1}^{\infty} f_{a(T_j+t)}(x)\mathrm{d}x \tag{17}$$

in which $T_0 = 0$.

The probability density function $f_{a(T_j+t)}(x)$ of the crack size at any service time $\tau = T_j + t\,(j = 0,1,2,\ldots)$ can be derived with the following results[9,10]:

A) In the first service interval ($j = 0$):

The solution $p(i,\tau)$ is identical to Eq. (14) in which $\tau \le T_1$ and

$$f_{a(t)}(x) = A_1(t;j) = f_{a(0)}[y_1(t)]I_1(t) \tag{18}$$

in which

$$y_1(t) = W(x,t), \qquad I_1(t) = [\mathrm{d}y_1(t)/\mathrm{d}t]/[\mathrm{d}x/\mathrm{d}t] \tag{19}$$

where $W(x,t)$ is the crack growth master curve [see Eq. (11)].

B) After the first service interval ($j = 1,2,\ldots$):

$$f_{a(T_j+t)}(x) = A_{j+1}(t;j) + \sum_{k=1}^{j} G(j-k+1)A_k(t;j) \tag{20}$$

$$A_k(t;j) = \left\{\prod_{m=1}^{k-1} F_D^*[y_m(t;j)]\right\} f_{a(0)}[y_k(t;j)]I_k(t;j) \tag{21}$$

$$k = 2,3,\ldots,j+1$$

$$\left.\begin{aligned} y_m(t;j) &= y_1\left[t + \sum_{n=j-m+2}^{j} \tau_n\right] = W\left[x, t + \sum_{n=j-m+2}^{j} \tau_n\right] \\ I_m(t;j) &= [\mathrm{d}y_m(t;j)/\mathrm{d}t]/[\mathrm{d}x/\mathrm{d}t] \end{aligned}\right\} \tag{22}$$

In Eq. (19), the crack size $y_I(t)$ and the slopes $dy_I(t)/dt$ and dx/dt are obtained from the master curve as shown in Fig. 4a. Similarly, the crack size $y_m(t;j)$ and the slopes $dy_m(t;j)/dt$ and dx/dt appearing in Eqs. (20-22) are also determined from the master curve, as shown in Fig. 4b.

Furthermore, $G(j)$ $(j=1,2,...)$ appearing in Eq. (20) represents the probability of detecting a crack of any size at the jth inspection

$$G(j) = \int_0^\infty g_j(x)\,dx \tag{23}$$

in which $g_j(x)dx$ is the probability of detecting a crack size in $(x,x+dx)$ [see Eq. (5)]

$$g_j(x) = f_{a(T_{j-1}+\tau_j)}(x) F_D(x) \tag{24}$$

where $T_0=0$ and $j=1,2,...$

E. Repaired Cracks and Structural Reliability

The probability of detecting (and repairing) a crack size between z_1 and z_2, denoted by $q_j(z_1,z_2)$, at the jth inspection $(j=1,2,...)$, i.e., at T_j flight hour, follows from Eqs. (20) and (24) as

$$q_j(z_1,z_2) = \int_{z_1}^{z_2} g_j(x)\,dx \tag{25}$$

Clearly, the probability of repairing (or detecting) a crack size greater than z_1 at the jth inspection is $q_j(z_1,\infty)$, which can be computed from Eq. (25). Furthermore, it follows from Eqs. (23-25) that $G(j)=q_j(0,\infty)$.

If a_{cr} denotes the critical crack size associated with the maximum design load P_{XX}, then the probability of failure $P_i(T_{j-1},T_j)$ of a detail in the ith stress region in the jth service interval (T_{j-1},T_j), see Fig. 1, is given by

$$P_i(T_{j-1},T_j) = \int_{a_{cr}}^\infty f_{a(T_{j-1}+\tau_j)}(x)\,dx \tag{26}$$

The probability of failure of the entire durability critical component, consisting of m stress regions, in the service interval $(0,T_j)$ can be shown as

$$P_f(0,T_j) = 1 - \prod_{i=1}^{m} \left\{ \prod_{\ell=1}^{j} \left[1 - P_i(T_{\ell-1},T_\ell) \right]^{N_i} \right\} \tag{27}$$

III. Numerical Example

The economic life of a hypothetical durability critical component of a fighter aircraft is considered for demonstrative purpose. For simplicity, the component is assumed to consist of only one stress region [i.e., $m=1$ in Eq. (3)] having 100 identical fastener holes where the maximum stress level of the fighter spectrum to each fastener hole is $\sigma_{max}=34$ ksi. Cracks are assumed to occur at fastener holes only.

Test data of TTCI at 0.03 in. for fastener holes with no load transfer, Winslow drilled with proper drilling technique,[8] are presented in Fig. 2a as circles. The three-parameter Weibull distribution [see Eq. (6)] is used to best fit the data as shown by the solid line where $\alpha=4.86$, $\beta=14{,}960$ h (50% confidence level), and $\epsilon=1312$ h. Furthermore, $\beta_\gamma=14{,}240$ h for $\gamma=95\%$ confidence level. The crack growth master curve and the crack growth rate are presented in Fig. 3. It is observed from Fig. 3b that for the crack size smaller than 0.03 in., Eq. (7) is valid (straight line) with $Q=0.925\times10^{-3}$, $b=1.2165$, and $c=b-1=0.2165$. Data of the EIFS are obtained from the backward extrapolation using Eq. (7), and they are presented in Fig. 2b as circles. The Weibull compatible distribution of EIFS given by Eq. (8) is also plotted in Fig. 2b as a solid curve and a dashed curve, respectively, associated with β and β_γ for $\gamma=95\%$ confidence level.

Without inspection and repair maintenance procedure, the probability $p(i,\tau)$ that a detail will have a crack size larger than x_1 at any service time τ is computed from Eq. (14). The average percentage of cracked details $\bar{L}(\tau)/N^*$ [see Eqs. (3-4)] exceeding a certain crack size x_1 (abscissa) is plotted in Fig. 5a as a function of service time τ in flight hours. For instance, at 8000 h, the average percentage of cracked details exceeding 0.03 and 0.05 in. are 2.0 and 0.8%, respectively. As expected, $\bar{L}(\tau)/N^*$ increases as the service time τ increases. Figure 5a is referred to as the average crack exceedance curve with 50% confidence. When β is replaced by β_γ for $\gamma=95\%$ confidence level, the resulting average crack exceedance curve is displayed in Fig. 5b. A comparison of Figs. 5a and 5b indicates that the average percentage of crack exceedance $\bar{L}(\tau)/N^*$ increases when a higher confidence level is considered.

When the total number of details (fastener holes) is large, the distribution of $N(i,\tau)$, which is binomial [see Eq. (1)], and the distribution of the total number of details having a crack size exceeding x_1, $L(\tau)$, can be approximated by the normal distribution with means and variances given by Eqs. (2) and (4). Consequently, the average crack exceedance curves presented in Fig. 5 represent the crack exceedance with 50% probability. The crack exceedance with any probability

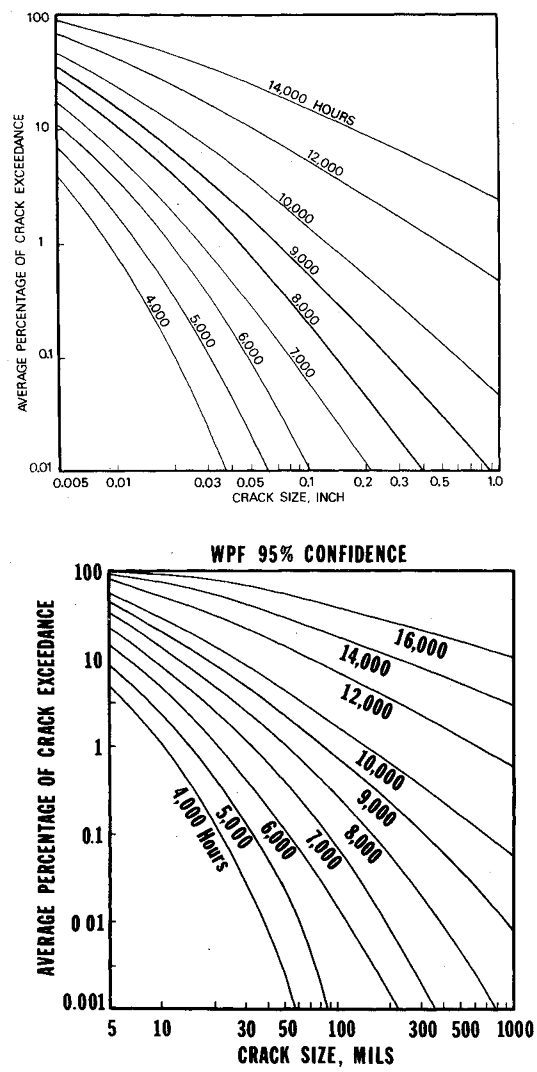

Fig. 5 Average crack exceedance curves: a) 50% confidence level, b) 95% confidence level.

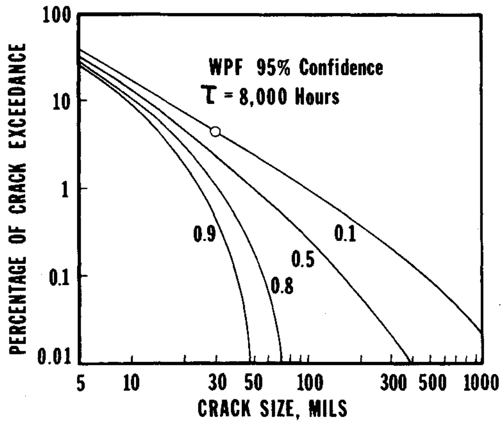

Fig. 6 Crack exceedance curves with 95% confidence level at 8000 flight hours for various exceedance probabilities.

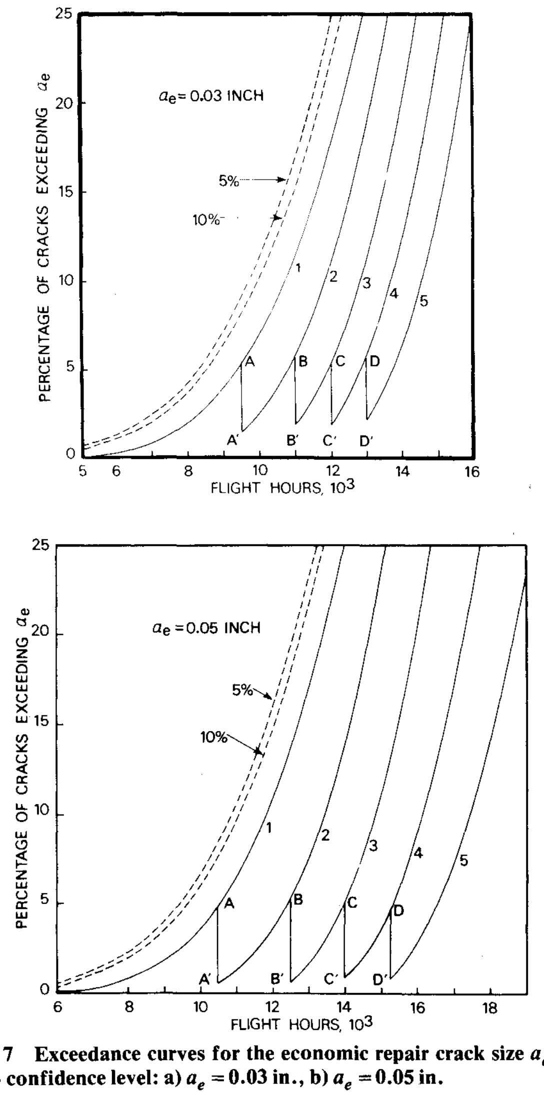

Fig. 7 Exceedance curves for the economic repair crack size a_e with 50% confidence level: a) $a_e = 0.03$ in., b) $a_e = 0.05$ in.

can be computed using the normal distribution for approximation.

For simplicity in presentation, the service time of $\tau = 8000$ flight h is considered. The percentage of cracks $L(\tau)/N^*$ exceeding the crack size x_1 (abscissa) is plotted in Fig. 6 for various exceedance probability with $\gamma = 95\%$ confidence level. For instance, at $\tau = 8000$ h, the probability is 0.1 that 4.51% of the total cracks will exceed 0.03 in. with 95% confidence level, as indicated by a circle. In other words, at 8000 h, 4.51% of the total cracks will exceed 0.03 in. with 0.1 probability and 95% confidence. The exceedance curves associated with various probabilities and γ% (such as 50%) confidence level can similarly be constructed.[9]

If 0.03 in. is used as the economic repair crack size, then the average percentage (i.e., with 50% probability) of the cracked details exceeding $a_e = 0.03$ in. is plotted as a function of the service time, as shown by Curve 1 of Fig. 7a for 50% confidence level. This curve, in fact, can be constructed from Fig. 5a by drawing a vertical line passing through 0.03 in. The exceedance curves for the economic crack size with probability 5 and 10%, respectively, are also computed [see Eqs. (2-4)] and plotted in Fig. 7a as dashed curves. For instance, at 8000 h, there are 2% of cracks exceeding a_e with 50% probability (Curve 1), 3.77% of cracks exceeding a_e with 10% probability, and 4.27% of cracks exceeding a_e with 5% probability, as observed from Fig. 7a. It is observed from Fig. 7a that Curve 1 increases rapidly after 8000 flight h.

If 5% of the cracked details exceeding 0.03 in. with probability 50% is considered as an economical limit, then the possible economic life is observed from Curve 1 to be 9400 flight h. When the economic repair crack size a_e is 0.05 in., one can construct in a similar fashion the exceedance curve with probabilities 50% (Curve 1), 10% (dashed curve), and 5% (dashed curve), respectively, as shown in Fig. 7b. The possible economic life in this case is found to be 10,500 flight h, if 5% of cracks exceeding 0.05 in. with probability 50% is considered as an economic limit.

To demonstrate the effect of maintenance procedures on the economic life, an NDI procedure with a high-resolution capability is assumed. The detection probability $F_D(x)$ is expressed in the following[14-16]: $F_D(x) = 0$ for $x < a_1$, $F_D(x) = [(x - a_1)/(a_2 - a_1)]^{m_1}$ for $a_1 \leq x \leq a_2$, and $F_D(x) = 1$ for $x > a_2$, where $a_1 = 0.01$ in., $a_2 = 0.1$ in. and $m_1 = 0.5$.

For $a_e = 0.03$ in., the first inspection and repair is performed at $T_1 = 9500$ h. The average exceedance curve (i.e., 50% probability) is obtained by use of Eqs. (20-24) and plotted in Fig. 7a as Curve 2. Owing to the repair procedures, there is a sudden drop from A to A′ (i.e., Curve 1 to Curve 2) at T_1, as indicated. According to Curve 2 of Fig. 7a, the possible economic life (5% allowable exceedance over a_e with 50% probability) is now extended to 11,000 flight h. If the second inspection and repair is further performed at $T_2 = 11{,}000$ h, then the resulting average exceedance curve is computed from Eqs. (20-24) and plotted as Curve 3. The economic life is thus extended to 12,000 h. In a similar manner, additional inspections and repair maintenance procedures are performed, respectively, at $T_3 = 12{,}000$ and $T_4 = 13{,}000$ h, and the results are plotted in Fig. 7a as Curves 4 and 5, respectively. It is observed from Fig. 7a that there is a sudden drop of the crack exceedance at each inspection and repair maintenance, which is exactly the average percentage of cracks repaired (or detected) during each maintenance.

Thus, if the component is subjected to a scheduled inspection and repair maintenance at T_1, T_2, T_3, and T_4, then the average crack exceedance curve will follow Curve 1 up to A and follows the path A→A′→B→B′→C→C′→D→D′→Curve 5 (see Fig. 7a). For the case where the economic repair crack size is 0.05 in., the inspection and repair maintenance procedures are performed at $T_1 = 10{,}500$, $T_2 = 12{,}500$, $T_3 = 14{,}000$, and $T_4 = 15{,}250$ h, respectively. The resulting average crack exceedance curves are presented in Fig. 7b.

The average percentage $q_j(z_1, \infty)$ [see Eqs. (24-25)] of details with a crack size larger than z_1, which are repaired at the jth inspection, is plotted in Fig. 8 for $j = 1,2,3,4$ for the case where $a_e = 0.03$ in. These curves are referred to as the average exceedance curves for repaired details. For instance, there are 1% of details with a crack size greater than 0.1 in. repaired at $T_1 = 9500$ h, as indicated by a circle on Curve 1. These exceedance curves indicate, on the average, the number of details as well as their corresponding crack size that will be

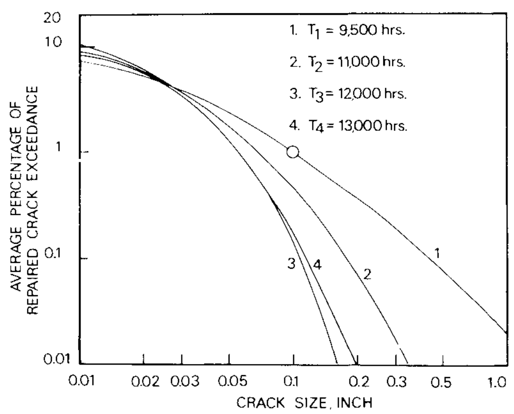

Fig. 8 Average exceedance curves for repaired details.

repaired during the inspection and repair maintenance. They can be used to evaluate the cost of repair as indicated by Eq. 5.

It is observed from Fig. 7 that the effect of inspection and repair maintenance procedures on the economic life is not significant even using a high-resolution NDI procedure. It is found that for other NDI techniques with less detection capability, the effect is practically minimal. The reasons are given as follows: 1) to extend the economic life, the NDI procedure should be able to detect with high probability the crack size below the economic repair crack size 0.03 in. in order to prevent cracks from propagating beyond a_e. Current NDI techniques may not achieve such a capability with comparable cost, and 2) as soon as the crack exceedance curve over a_e starts to increase rapidly (see Fig. 7), the majority of the crack population is already in the vicinity of a_e, and hence a significant extension of the economic life implies an extensive repair such that the cost of repair may be uneconomical.

IV. Conclusion and Discussion

A statistical method for predicting the economic life of durability critical components of aircraft structures is presented. The approach allows for the determination of the economic life using either one of the two criteria: 1) a rapid increase of the crack population exceeding the economic repair crack size, and 2) a rapid increase of the maintenance cost including the costs of inspection and repair. While the inspection and repair maintenance procedure has a significant effect on aircraft structural reliability and safety,[14-18] its effect in extending the economic life is shown to be limited.

While the second criterion of the economic life has been formulated, it is not demonstrated in the numerical example although relevant statistics such as $g_j(x)$ have been computed. This is due to the lack of realistic informaiton of the field inspection cost and the costs of repairing various crack sizes. It appears that the first criterion of the economic life may be more appropriate at the preliminary design stage to satisfy the durability requirement. Although the present analysis includes the structural reliability as represented by Eq. (27), it will be demonstrated elsewhere. The present approach is applicable to cracks which occur at radii or shear webs with holes.

There are circumstances where the usage or mission of aircraft may be changed during its design life. This, however, can easily be taken care of in the present analysis. For instance, if the usage change occurs at t_c flight hours, then the new crack growth master curve (see Fig. 3a) corresponding to the new usage should be used in the analysis after t_c and all the analysis procedures remain the same.

In the present approach, the crack growth master curve under spectrum loading is considered deterministic, i.e., the statistical variability of crack growth parameters are neglected. This is a reasonable approximation, since the statistical variability of the crack growth parameters is smaller than that of the EIFS, and hence the statistical variability of the exceedance curves (or economic life) is essentially due to that of the EIFS. Such an approximation simplifies the entire analysis tremendously. It should be emphasized, however, that the statistical variability of the crack growth parameters is important and should be accounted for in the damage tolerant analysis where the initial flaw size is assumed to be deterministic, i.e., 0.005 in.

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